

STABILITY OF SHAPES HELD BY SURFACE TENSION AND SUBJECTED TO FLOW

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Abstract

Results of three problems are summarized in this contribution. Each involves the fundamental capillary instability of an interfacial bridge and is an extension of previous work. The first two problems concern equilibrium shapes of liquid bridges near the stability boundary corresponding to maximum length (Plateau-Rayleigh limit). For the first, a previously formulated nonlinear theory to account for imposed gravity and interfacial shear disturbances in an isothermal environment [1,2] is quantitatively tested in experiment. For the second problem, the liquid bridge is subjected to a shear that models the effect of a thermocapillary flow generated by a ring heater in a liquid encapsulated float-zone configuration[3]. In the absence of gravity, this symmetric perturbation can stabilize the bridge to lengths on the order of 30% beyond the Plateau-Rayleigh (PR) limit, which is on the order of heretofore unexplained shuttle observations. The third problem considers the dynamics of collapse and pinchoff of a film bridge (no gravity) --- what happens in the absence of stabilization. Here we summarize experimental efforts to measure the self-similar cone-and-crater structure predicted by a previous theory[4].

Introduction

A liquid/gas or liquid/liquid interface is shaped by surface tension whenever surface area is large relative to volume (small physical length) or when gravity is reduced relative to the surface force (small capillary length). The stability of such an interface is important to a variety of earth-based applications, to float-zone experiments in a space shuttle and to successful liquid management in a space laboratory, more generally.

Overview

Deformable interfaces of finite extent are of interest. Attention is restricted to axisymmetric shapes and pinned contact lines. Axisymmetric disturbances are the most dangerous for the axisymmetric shapes considered. These may be classified according to number of relevant length scales.

With only surface tension acting, an incompressible fluid mass of prescribed volume V takes the shape of a sphere. The sphere is characterized by a single length-scale, say $V^{1/3}$. If the fluid mass contacts a solid circular disk of radius R , then its equilibrium shapes (pieces of spheres) are characterized by length scales R and $V^{1/3}$. If, in addition, gravity acts on the mass (force/volume ρg) then the shapes depend on capillary length $(\sigma/\rho g)^{1/2}$ as well as on R and $V^{1/3}$. These figures are known as the sessile drop, bubble or pendant drop, depending on the orientation of gravity and density contrast. Three lengths are relevant, alternatively, if, in the absence of gravity, the mass contacts two solid disks, arranged coaxially and separated by length ℓ . This is known as the liquid bridge. Droplets and bridges can also be exposed to a flow. Suppose p_c represents a typical pressure gradient. Then, an additional relevant length scale enters, $\sigma/p_c R$, say. And so forth. Ratios of relevant lengths form the dimensionless control parameters of the problem. Interfacial bridges are characterized by two contact lines and at least two control parameters.

The number of contact lines is related to potential instabilities of the shapes. The sphere is always stable. Shapes with a single contact line (spherical bubbles and pendant or sessile droplets) suffer only turning point instabilities (codimension 0 bifurcations). On increasing ℓ , liquid bridges, on the other hand, undergo pitchfork bifurcations that can be unfolded with two control parameters (codimension 2). Singularity theory [5] provides the mathematical framework for the experimental results presented in the first liquid bridge problem. The interacting symmetries of pitchfork and imposed disturbance provide the framework for the second liquid bridge problem. The thin film bridge is a liquid bridge with zero net curvature ($\kappa_1 + \kappa_2 = 0$). In that problem, the dynamics of interest start at the instability occurring at a turning point bifurcation.

The influence of flow on capillary instability can be understood through the normal stress balance across the interface. In the absence of motion and body force and in an isothermal environment, this reduces to the Young-Laplace equation,

$$[p] = \sigma(\kappa_1 + \kappa_2) \quad (1)$$

where $[p]$ is the jump in pressure across the interface and κ_1 and κ_2 are the principal curvatures of the math-

emational surface. Thus, the axial pressure gradient $[p_z]$ can be viewed as having two contributions, each from a curvature gradient. If gravity acts coaxially, there is an additional constant contribution proportional to ρg . Suppose motion arises from an imposed interfacial shear. Although the motion is driven through the tangential stress balance, it influences the shape as an axial pressure gradient in the normal stress balance. Therefore, the free boundary problem for the interface can be solved with the following strategy. Guess an interface shape, solve for the flow field to obtain the flow contribution to the pressure gradient, solve the normal stress balance for a corrected shape, and so forth. This approach also works for the nonisothermal case provided the coupling between thermal and velocity fields occurs only at the interface (ie. small Peclet number). Our analyses of liquid bridges follows this approach. A bifurcation equation is derived from a functional equation representing the normal stress balance. The steady states of this equation are studied.

Liquid bridge: shear and gravity effects

The bridge is subject to gravity and is immersed in a pipe flow, with both perturbations acting coaxially, as sketched in figure 1. We have previously established that although each perturbation on its own is destabilizing, they can stabilize by acting in concert[6]. This is the fingerprint of a nonlinear effect. Here we summarize experimental results that probe the neighborhood of the PR limit. Figure 2 plots the deflection ϵ of the interface from cylindrical against the flow rate Q . The solid lines are the predictions of the bifurcation equation derived from a normal stress balance that takes account of motion of liquids on both sides of the interface[7]. The symbols represent measurements taken over a range of strengths of gravity and flow rate, lengths and volumes. Here Bond number is defined as $B \equiv r_o^2 \rho g / \sigma$, scaled length $L \equiv \ell / r_o$ and volume imperfection v is the deviation of the volume from cylindrical, scaled by the cylindrical volume.

The lengths can be ordered from 10% to 3% short of the PR limit. Several trends are discernible. For equal lengths, the slopes are comparable and the magnitude of slope increases with increasing length. The position of the intercepts depend on volume and Bond number. The region between the turning points gives the window of stable states. This is a true window (upper and lower limits) for quiescent shapes that bulge down ($B > 0$). The window narrows as the length increases. The the-

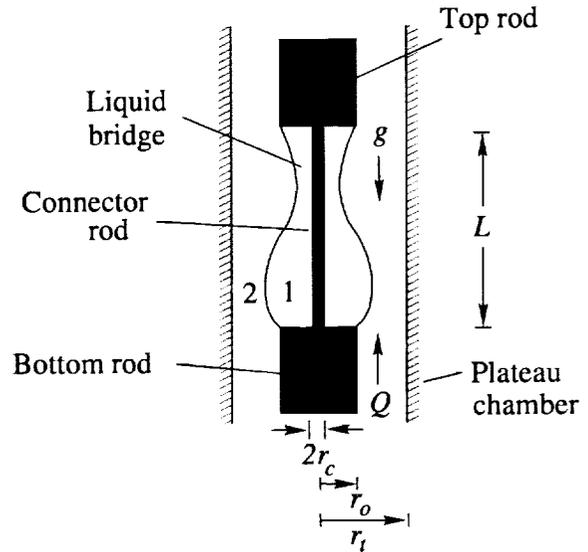


Figure 1: Sketch of a two-fluid system.

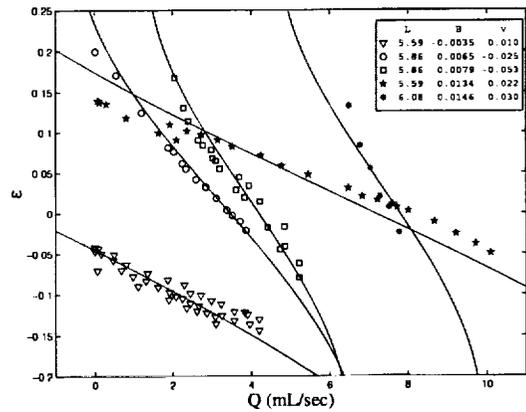


Figure 2: Surface deflection as a function of flow rate. Symbols represent experiment and solid lines are two-fluid theory [7].

ory is not only able to account for trends in the data, but it is capable of quantitative prediction. Figure 2 further suggests that the regime of validity of theory is considerable, especially as regards to length. Theory and experiment are in tight agreement except closest to the stability limit where experimental limitations of temperature control seem to preclude reliable measurements [7]. This explains the absence of data beyond the PR limit ($L = 2\pi$).

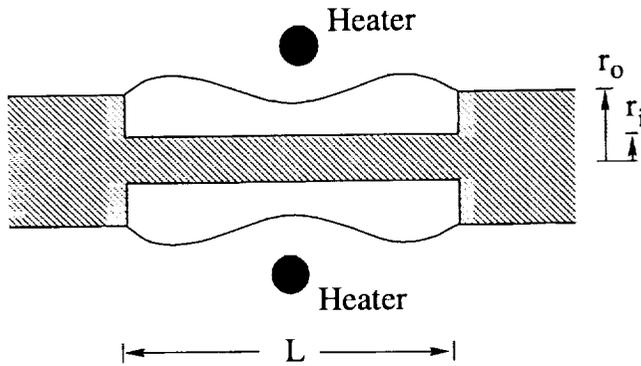


Figure 3: Sketch of a liquid float-zone in zero gravity.

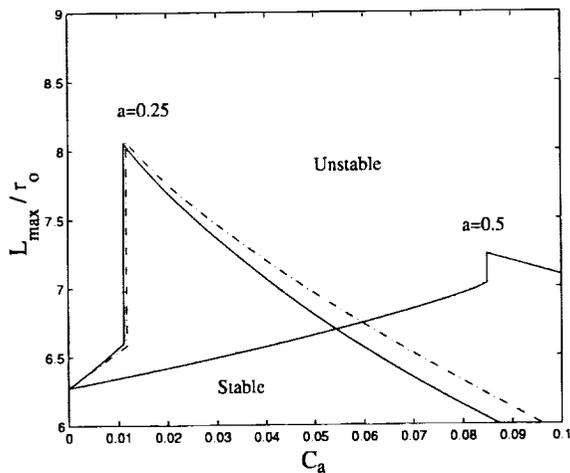


Figure 4: Maximum bridge length versus capillary number at $a=0.25$ and $a=0.5$ where $a \equiv 1 - r_i/r_o$. Dashed line represents results from perturbation method and solid lines are from computation.

Liquid bridge: encapsulated float-zone model

The second problem is motivated by observations of extra-long float zones in the Liquid Encapsulated Melt Zone (LEMZ) materials science experiment on STS57[8]. The float zone is modeled as a liquid bridge (no gravity) whose normal stress balance is influenced by pressure gradients induced by thermocapillarity. In contrast to the isothermal problem where the imposed shear is unidirectional, symmetry of the full float zone (ring heater) generates a shear symmetric about the midplane. In figure 3, the solid center rod that makes the bridge an annulus models an unmelted core or a viscosity that varies with temperature. The symmetric pressure disturbances can interact significantly with the PR pitchfork. Figure 4 plots stability limit against strength of thermocapillary flow where $Ca \equiv -\alpha\Delta T/\bar{\sigma}$ and $\sigma \equiv \bar{\sigma} + \alpha(T - \bar{T})$ and T is temperature. Figure 4 predicts stabilizations of 30-40%, depending on the extent of the solid core, offering an explanation for observations (largely qualitative in nature).

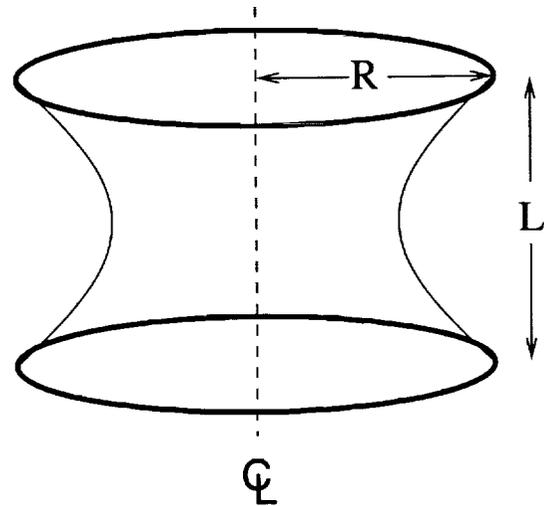


Figure 5: Schematic of soap film experiment.

Film Bridge: collapse and pinchoff

Figure 5 shows a sketch of the thin film stretched between two circular contact lines. The collapse is driven by capillary instability and resisted by the inertia of the surrounding fluid (air). Of interest is the prediction i) that both principal curvatures κ_1 and κ_2 diverge by a $t^{-2/3}$ scaling law as time t approaches the instant of

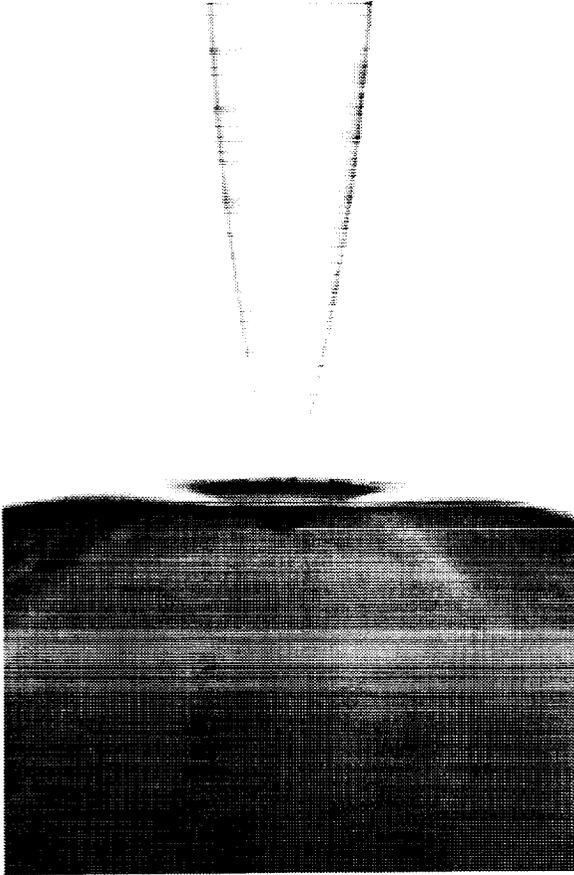


Figure 6: Photograph of cone-and-crater structure (shortly after detachment).
 topological change and ii) that a cone-and-crater pinch-off structure with unique angles appears[4]. Figure 6 shows a close-up photograph of the cone-and-crater structure, just after detachment of the cone from the crater. The key prediction that the crater forms before detachment has been verified in experiment. Experiments to accurately determine the angles are underway. This prototype problem is important since aspects of the inertially-dominated film pinchoff are likely to be common to inertial pinchoff in liquid systems.

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